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Transverse acoustic oscillations in cylindrical chambers with the radial temperature gradient

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Abstract. In this article the transverse acoustic oscillations in cylindrical chambers with a radial temperature gradient are considered. Expression for the velocity potential is calculated. This expression describes the free transverse oscillations in cylindrical chamber with radial temperature gradient.

1. Introduction

The interest in acoustic oscillations of the gas is maintained because of the wide spread them in various fields of science and technology. Many technical devices are some bounded by solid walls, volume, that is are these or other resonators. For example, the basic elements of gas turbines, steam generators, aircraft engines, gas lasers, plasma generators, and others. Therefore, the study of acoustic oscillations in the resonant modes of operation such devices is a major challenge. When using all these devices, oscillatory processes occur in a non-uniform temperature field.

Longitudinal acoustic oscillations with axial temperature gradient in a large number of works are studied [1, 2]. In this distribution of transverse of acoustic oscillations in the tube with a radial temperature gradient has not been considered. An example of a device with a radial temperature gradient can serve as generators of low-temperature plasma – plasma generators, high-power gas lasers, spectrum analyzers, and others [3-5]. Various methods of controlling the glow discharge parameters have been considered in [6-11]. In particular, excitation of transverse of acoustic oscillations in the discharge chamber can intensified heat transfer in the radial direction compared with the longitudinal acoustic oscillations [12]. To solve the self-consistent problem on the influence of the transverse acoustic oscillations on the plasma glow discharge is necessary to consider the reverse effect of the discharge on the characteristics of transverse acoustic waves. In this paper we consider the influence of radial temperature gradient on the parameters of the transverse acoustic oscillations in the chambers with a cylindrical geometry.

2. Transverse acoustic oscillations in cylindrical chamber with the radial temperature gradient

For the description of acoustic oscillations in the tube with a radial temperature gradient write the equations of motion and continuity in the cylindrical coordinate system.

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u\frac{\partial u_r}{\partial x}-\frac{u_\theta^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{1}$$

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u\frac{\partial u_\theta}{\partial x}-\frac{u_r u_\theta}{r}=-\frac{1}{\rho}\frac{\partial p}{r\partial \theta}, \tag{2}$$

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$$\frac{\partial u_x}{\partial t}+u_r\frac{\partial u_x}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_x}{\partial \theta}+u\frac{\partial u_x}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{3}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u_x)}{\partial x}+\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta}=0, \tag{4}$$

where x, r, θ – longitudinal, radial and angular coordinates, respectively, u_x, u_r, u_θ – the speed of the projection of the gas corresponding to the longitudinal, radial and angular coordinates, ρ – density, p – pressure.

Represent the flow parameters as the stationary and pulsating parts

$$\begin{aligned} u_r &= \bar{u}_r + u'_r, \\ u_\theta &= \bar{u}_\theta + u'_\theta, \\ u_x &= \bar{u}_x + u'_x, \\ \rho &= \rho_0 + \rho', \\ p &= p_0 + p'. \end{aligned} \tag{5}$$

Due to the radial temperature gradient and the constant pressure in the tube $\rho_0=\rho_0(r)$.From (1-4) and (5) we obtain a system of equations, which is linearized by taking into account the following equation $\bar{u}_r=\bar{u}_\theta=0$

$$\frac{\partial u'_r}{\partial t}+\bar{u}_x\frac{\partial u'_r}{\partial x}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial r}, \tag{7}$$

$$\frac{\partial u'_\theta}{\partial t}+\bar{u}_x\frac{\partial u'_\theta}{\partial x}=-\frac{1}{\rho_0 r}\frac{\partial p'}{\partial \theta}, \tag{8}$$

$$\frac{\partial u'_x}{\partial t}+\bar{u}_x\frac{\partial u'_x}{\partial x}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial x}. \tag{9}$$

$$\frac{\partial \rho'}{\partial t}+\bar{u}_x\frac{\partial \rho'}{\partial x}+u'_r\frac{\partial \rho_0}{\partial r}+\rho_0\left(\frac{\partial u'_x}{\partial x}+\frac{u'_r}{r}+\frac{\partial u'_r}{\partial r}+\frac{1}{r}\frac{\partial u'_\theta}{\partial \theta}\right)=0. \tag{10}$$

Using expressions $\rho_0=\gamma\frac{p_0}{c^2(r)}$, $\rho'=\frac{p'}{c^2(r)}$, $c(r)=\sqrt{\gamma R_g T(r)}$ [13], where γ – adiabatic index, c – velocity of sound, R_g – gas constant, T – temperature, rewrite the equation (10)

$$\frac{\partial p'}{\partial t}+\bar{u}_x\frac{\partial p'}{\partial x}+\gamma p_0\left(\frac{\partial u'_x}{\partial x}+\frac{u'_r}{r}+\frac{\partial u'_r}{\partial r}+\frac{1}{r}\frac{\partial u'_\theta}{\partial \theta}-\frac{2u'_r}{c}\frac{\partial c}{\partial r}\right)=0. \tag{11}$$

We express the velocity through the velocity potential [14] by means of relations

$$u'_x=\frac{\partial \varphi}{\partial x},\ u'_r=\frac{\partial \varphi}{\partial r},\ u'_\theta=\frac{1}{r}\frac{\partial \varphi}{\partial \theta}. \tag{12}$$

From the equations (7) and (9) we obtain

$$p'=-\rho_0\left(\partial\varphi/\partial t+\bar{u}_x\partial\varphi/\partial x\right) \tag{13}$$

Substitute (12), (13) into (11) and obtain the wave equation for the velocity potential

$$c^2\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial \theta^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}-\frac{2}{c}\frac{\partial\varphi}{\partial r}\frac{\partial c}{\partial r}\right)-2\bar{u}_x\frac{\partial^2\varphi}{\partial x\partial t}-\bar{u}_x^2\frac{\partial^2\varphi}{\partial x^2}=\frac{\partial^2\varphi}{\partial t^2} \tag{14}$$

The velocity potential must satisfy the following conditions

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r=r_0}=0,\ \varphi(\theta+2\pi)=\varphi(\theta),\tag{15}$$

where r_0 – the radius of the tube.

Suppose that the average flow in the tube is missing $\bar{u}_x=0,$. Then (14) takes the form

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}-\frac{2}{c}\frac{\partial c}{\partial r}\frac{\partial\varphi}{\partial r}=\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}.\tag{16}$$

Consider the mode of harmonic oscillations. In this case, the velocity potential may be represented as $\varphi=\bar{\varphi}(r,\theta,x)e^{i\omega t}$, where ω – angular frequency. Velocity potential due the boundary condition (15) is a periodic function of angular coordinate θ. Therefore, it can be expanded in a trigonometric Fourier series. Then we obtain

$$\varphi=\sum_0^\infty\varphi_m(r,x)\cos m\theta e^{i\omega t},\tag{17}$$

where $m=1,2,\ldots$.

Substituting (17) into (16) obtain

$$\sum_0^\infty\left(\frac{\partial^2\varphi_m}{\partial x^2}+\frac{\partial^2\varphi_m}{\partial r^2}-\frac{m^2}{r^2}\varphi_m+\frac{1}{r}\frac{\partial\varphi_m}{\partial r}-\frac{2}{c}\frac{\partial c}{\partial r}\frac{\partial\varphi_m}{\partial r}+\frac{\omega^2}{c^2}\varphi_m\right)\cos m\theta=0.\tag{18}$$

To solve the equation (19) using Fourier method represent $\varphi_m=R_m(r)X_m(x)$. This results in two equations

$$\frac{\partial^2X_m}{\partial x^2}+\lambda X_m=0,\tag{20}$$

$$\frac{\partial^2R_m}{\partial r^2}+\left(\frac{1}{r}-\frac{2}{c}\frac{\partial c}{\partial r}\right)\frac{\partial R_m}{\partial r}+\left(\frac{\omega^2}{c^2}-\frac{m^2}{r^2}-\lambda\right)R_m=0,\tag{21}$$

Where λ – constant.

The general solution of the equation (20) is

$$X_m=A\cos\left(\sqrt{\lambda}x\right)+B\sin\left(\sqrt{\lambda}x\right)\tag{22}$$

Suppose that the tube is closed at both ends. Then boundary conditions are

$$\left.\frac{\partial X_m}{\partial x}\right|_{x=0}=\left.\frac{\partial X_m}{\partial x}\right|_{x=L}=0,\tag{23}$$

where L – length of the tube.

From (23) and (22) we obtain $B=0$ and $\lambda=\left(\frac{\pi n}{L}\right)^2$. The solution of equation (20) will take the final form

$$X_m=A\cos\left(\frac{\pi n}{L}x\right),.\tag{24}$$

where $n=0,1,2,\ldots$.

It is known that in the cylindrical discharge chambers of gas lasers, the temperature distribution along the radius is parabolic [6]. Then, since $c \sim \sqrt{T}$, the velocity of sound can be represented as

$$c=\sqrt{b-ar^2},\tag{25}$$

where a and b – constants.

Substituting (25) into (21) consider only the transverse oscillations. For this set $n=0$, $\lambda=0$ and obtain

$$\frac{\partial^2 R_m}{\partial r^2}+\left(\frac{1}{r}+\frac{4ar}{b-ar^2}\right)\frac{\partial R_m}{\partial r}+\left(\frac{\omega^2}{b-ar^2}-\frac{m^2}{r^2}\right)R_m=0. \qquad (26)$$

The solution of equation (26) can be written as

$$R_m=C_1r^mF\left(\alpha_1,\beta_1;\chi_1;\delta\right)+C_2r^{(-m)}F\left(\alpha_2,\beta_2;\chi_2;\delta\right), \qquad (27)$$

where C_1, C_2 – constants, F – hypergeometric function,

$$\alpha_1=\frac{1}{2}\left(\frac{-2a+\sqrt{a\left(4a+\omega^2+m^2a\right)}+ma}{a}\right), \qquad \beta_1=\frac{1}{2}\left(\frac{2a+\sqrt{a\left(4a+\omega^2+m^2a\right)}-ma}{a}\right),$$

$$\chi_1=1+m, \qquad \delta=\frac{r^2a}{b}, \qquad \alpha_2=\frac{1}{2}\left(\frac{-2a+\sqrt{a\left(4a+\omega^2+m^2a\right)}-ma}{a}\right),$$

$$\beta_2=\frac{1}{2}\left(\frac{2a+\sqrt{a\left(4a+\omega^2+m^2a\right)}+ma}{a}\right), \quad \chi_2=1-m.$$

Second term on the right side of (27) tends to infinity at $r=0$. From the condition of finiteness, it follows that $C_2=0$ and equation (27) takes the form

$$R_m=C_1r^mF\left(\alpha_1,\beta_1;\chi_1;\delta\right). \qquad (28)$$

Then solution of the wave equation (16), which takes into account only the transverse oscillations is

$$\varphi=Cr^mF\left(\alpha_1,\beta_1;\chi_1;\delta\right)\cos m\theta e^{i\omega t}, \qquad (29)$$

where $C=C_1A$.

3. Conclusions

Thus, the transverse acoustic oscillations in cylindrical chamber with a radial temperature gradient describes by the velocity potential, which defined by (29). Expression (29) can be used to determine the natural frequencies of transverse oscillations in various devices, operating environment which is the discharge plasma. In particular, in high-power gas lasers with diffuse cooling.

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